

Retake Exam - Analyse II - Wiskunde

Friday, July 6, 2018, 14.00-17.00

- Write your name and student ID in a **clearly readable** manner on each page.
- Every answer has to be motivated by a computation, explanation of reasoning or reference to the theory.
- Calculators may be used only if **non-graphical**.

This exam has *four* questions.

Question 1

Consider the vector field $F : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ given by

$$F(x, y) = (x + \sqrt{2}y, y) .$$

The curve \mathcal{C} is the path from $(0, 0)$ to $(0, 2)$ along the half ellips $\frac{1}{2}x^2 + y^2 = 2y$ with $x \geq 0$.

- (a) Compute the vector line integral

$$\int_{\mathcal{C}} F(r) \cdot dr$$

through direct calculation.

- (b) Compute the vector line integral in (a) again, but this time using the fact that (x, y) is conservative.
(c) Compute the vector line integral in (a) again, but this time using Green's theorem.

Question 2

Consider the vector field

$$F(x, y, z) = \left(z^2x, \frac{1}{3}y^3 + \tan(z), x^2z + 1 \right)$$

and the surface

$$\mathcal{S} = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1, z \geq 0\} \subset \mathbb{R}^3 ,$$

oriented such that the normal vector points upwards.

- (a) Compute $\operatorname{div}(F)$ and $\operatorname{curl}(F)$.
(b) Compute (in a convenient way) the flux of F through \mathcal{S} , that is, the vector surface integral

$$\int_{\mathcal{S}} F \cdot dS .$$

Please turn the page. There are more questions on the back!

Question 3

Consider the scalar function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ given by

$$f(x, y) = \frac{x^4 + y^4 - x^2 - y^2}{\sqrt{x^2 + y^2}}, \quad (x, y) \neq (0, 0), \quad f(0, 0) = 0,$$

and the domain

$$\mathcal{D} = \left\{ (x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq \frac{1}{3} \right\} \subset \mathbb{R}^2.$$

- (a) Is f differentiable on \mathbb{R}^2 ?
- (b) Determine critical points of f on \mathbb{R}^2 . *Hint: You do not have to classify them.*
- (c) Show that $(x, y) = \left(0, \frac{1}{\sqrt{3}}\right)$ is a saddle point for f on \mathbb{R}^2 .
- (d) Find all global maxima/minima of f on \mathcal{D} .

Question 4

Compute the integral

$$\int_1^{\sqrt{2}} \int_{-\frac{2}{\sqrt{3}}z}^{\frac{2}{\sqrt{3}}z} \int_{\frac{1}{2}x}^{\frac{1}{2}(x + \sqrt{4z^2 - 3x^2})} z e^{x(x-y)} e^{y^2} dy dx dz.$$